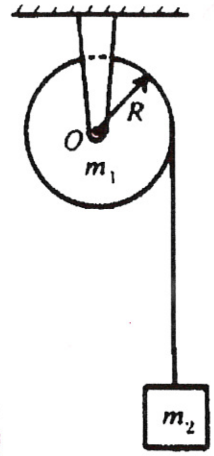
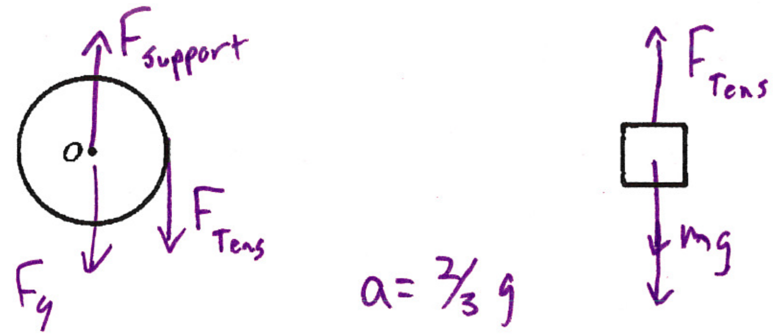
2. A pulley of mass $m_1 = M$ and radius R is mounted on frictionless bearings about a fixed axis through O. A block of equal mass $m_2 = M$, suspended by a cord wrapped around the pulley as shown above, is released at time $t = 0$. The acceleration of the block is measured to be $(2/3)g$ in an experiment using a computer-controlled motion sensor.



a. On the diagram below draw and identify all of the forces acting on the pulley and on the block.



$a = 2/3\, g$

should be bigger than F_{Tens}.

b. In terms of M, R, and g, determine each of the following.

i. The tension in the cord

For block:

$$\Sigma F = ma$$
$$mg - F_{Tens} = m \cdot 2/3\, g$$
$$F_{Tens} = 1/3\, mg$$

ii. The torque on the pulley

$$\tau = F r \cancel{\sin\theta}$$
$$= F_{Tens} \cdot R = 1/3\, mg \cdot R$$

iii. The angular acceleration of the pulley

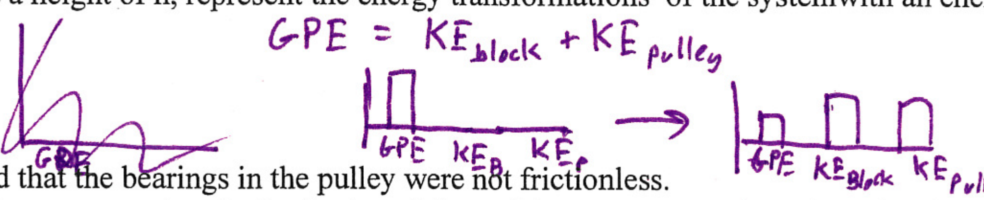
$$\alpha = a/R$$
$$= 2/3 \cdot g/R$$

ii. The rotational inertia of the pulley

$$\Sigma \tau = I\alpha$$
$$1/3\, mgR = I \cdot 2/3\, \frac{g}{\cancel{m}} R \rightarrow I = 1/2\, mR^2$$

c. If the block falls a height of h, represent the energy transformations of the systemwith an energy bar chart and equations.

$GPE = KE_{block} + KE_{pulley}$

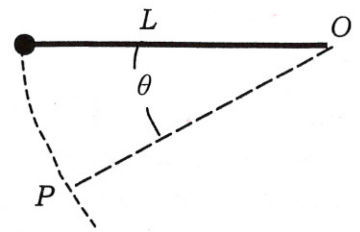


It is later determined that the bearings in the pulley were not frictionless.

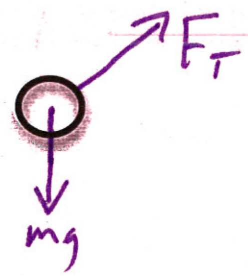
d. Assuming the values presented at the beginning of the problem are correct, how does the size of the real value of the rotational inertia of the pulley compare to the value determined in part b? Justify your answer.

I would actually be less. It would fall faster without friction, increasing α, so I would be less than what we calculated.

3. In an experiment, a pendulum consisting of a small heavy ball of mass m glued at the end of a rod of length L (negligible mass) is released from a horizontal position. The rod is pivoted at the other end O, but is free to rotate. When the ball is at point P, the rod forms an angle of θ with the horizontal as shown. The rotational mass of the rod-ball combination is mL^2.



a) The circle below represents the ball when it is at point P. Draw and label arrows representing the forces on the ball. The tail of each arrow should be placed at the point of application of the force.



b) The ball continues to fall until it reaches point Q, where the rod is vertical. At point Q, is the tension in the rod greater than, less than, or equal to what it was at point P? Justify your answer qualitatively without using equations.

Greater. Need to go in a circle, and going fastest at bottom. Higher speed needs greater centrip. accel., needs greater Tens.

c) The experiment is to be repeated with a new small ball, of mass $2m$, glued to the end of the rod of length L. Two students differ in their prediction for the rotational kinetic energy at point Q, and they come to you for help.

Student 1 claims, "The mass cancels out, so the speed of the ball will be the same as it was before, so the rotational kinetic energy will be the same."

Student 2 claims, "The different masses will cause different angular speeds at point Q, so the rotational kinetic energy will be different."

(i) For each of the students' arguments identify one correct portion of their reasoning.

1 Correct: Same speed

2 correct: diff KE

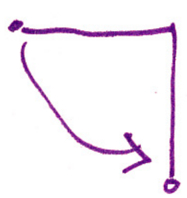
(ii) For each of the students' arguments identify one incorrect portion of their reasoning.

Speeds

1 incorrect: Same KE

2 incorrect: diff angular speeds.

(iii) Calculate the velocity at point Q for the new experiment. (Hint: use v=ωR to find v)



$\Delta h = L$

$\Delta GPE = \Delta KE$

$\cancel{m}gL = \frac{1}{2}\cancel{m}r^2\omega^2$

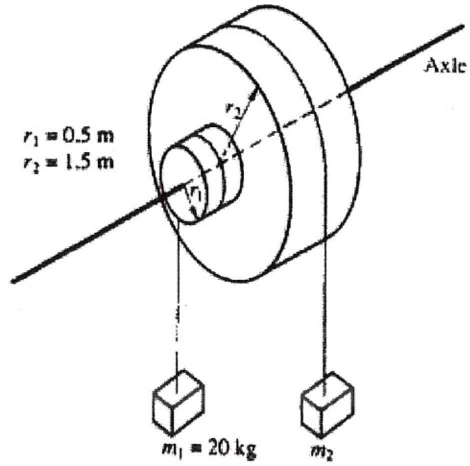
or

$GPE = \frac{1}{2}mv^2$

$v = \sqrt{2gh}$

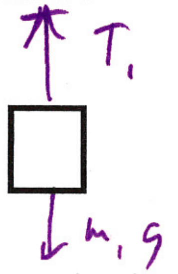
4.

The diagram shows two masses, m1 and m2, that are connected by light cables as shown in the diagram above. They are connected about the perimeters of two cylinders of radii r_1 and r_2, respectively, as shown in the diagram above with $r_1 = 0.5$ meter, $r_2 = 1.5$ meters, $m_1 = 20$ kilograms, and $m_2 = 25$ kilograms.

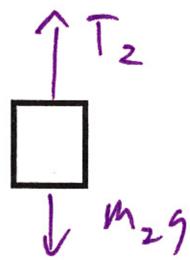


a. In the space below, draw a force diagram showing all of the forces acting on the masses.

mass 1



mass 2



b. i. Write a true statement comparing the torque of mass 1 to the torque caused by mass 2. In a sentence, justify your reasoning why this statement should be true.

$m_2 g \cdot r_2 > m_1 g \cdot r_1$

ii. What can you say about the rotational acceleration of cylinder 1 compared to the rotational acceleration of cylinder 2?

Must be same → glued together

c. When released the heavier block falls 50 cm in 4 s. Determine the magnitude of the blocks' acceleration?

$\cancel{2a\Delta x = v_f^2}$

$\Delta x = \frac{1}{2} a t^2$

$a = \frac{2\Delta x}{t^2} = \frac{1\,m}{16\,s^2} = 0.0625\ m/s^2$

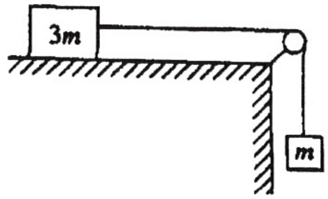
d. If you want the system to be in equilibrium, what value should you replace mass 2 with to achieve equilibrium?

Set

$m_1 g r_1 = m_2 g r_2$

$20 \cdot 0.5 = m_2 \cdot 1.5$

$\frac{10}{1.5} kg = m_2$



5. A block of mass $3m$ is placed on a frictionless table with a string over a frictionless pulley connecting it to a hanging block of mass m. Assume that the string has negligible mass and, for part (a), assume that the pulley also has negligible mass.

a) If the $3m$ mass had been placed on a table with some friction (but not enough to stop it accelerating) would the force of tension in the string attached to the hanging mass be greater, less than, or the same as if it was frictionless? Explain your answer qualitatively and use equations.

Greater. Would go slower, need to lower accel of m, so need greater T since

$mg - T = ma$

stays same

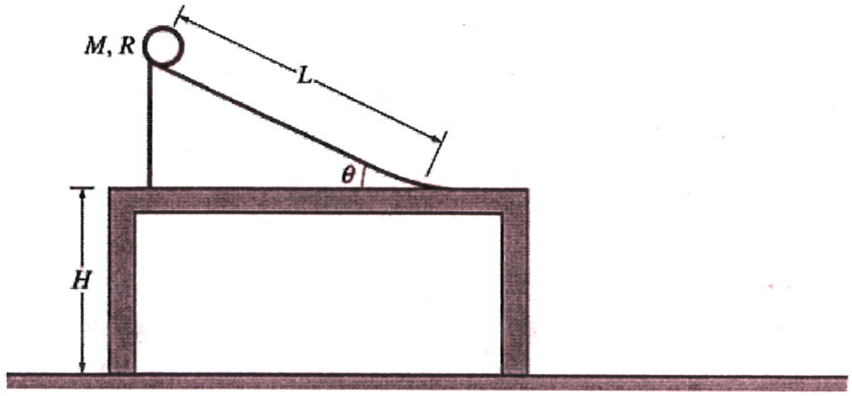
goes down

b) If the $3m$ mass had been placed on a frictionless table but the pulley were replaced by a pulley with significant rotational inertia (but not enough to stop it accelerating) would the force of tension in the string attached to the hanging mass be greater, less than, or the same as if it was frictionless? Explain your answer qualitatively, without the use of equations.

Greater. Would go slower, same explanation as before.

c) How does the acceleration of the hanging block in part b compare to the block without friction?

Less!



6.

A thin hoop of mass M, radius R, and rotational inertia MR^2 is released from rest from the top of the ramp of length L above. The ramp makes an angle θ with respect to a horizontal tabletop to which the ramp is fixed. The table is a height H above the floor. Assume that the hoop rolls without slipping down the ramp and across the table. Express

If the hoop is now replaced by a disk with the same mass M and radius R. How will the distance from the edge of the table to where the disk lands on the floor compare between the hoop and the disk? Explain in terms of energy and forces in two separate statements.

Energy: Hoop has greater rotational inertia, so more energy is used up in spinning it (rather than translating), so it will have lower x-velocity at bottom of ramp. Both will take same time to hit floor. Hoop can't go as far.

Forces: Since mass is same, F_g is same, and so must friction. Since forces are the same, and radius is same, torques must be equivalent. If $\Sigma\tau$ doesn't change, but I does, α must be different. α is smaller for ring b/c I is greater.

So...

Views From Above

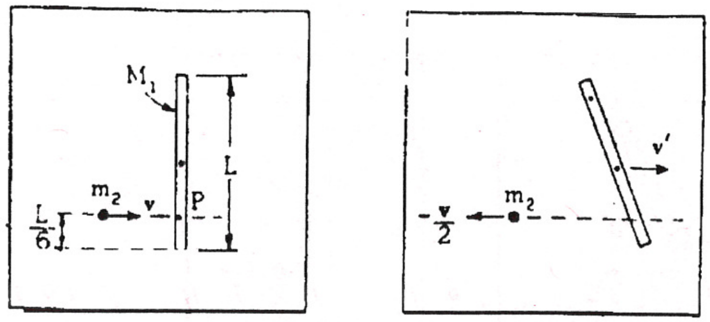


Figure I: Before

Figure II: After

8. A thin, uniform rod of mass M_1 and length L , is initially at rest on a frictionless horizontal surface. The moment of inertia of the rod about its center of mass is $(1/12)M_1L^2$. As shown in Figure I, the rod is struck at point P by a mass m_2 whose initial velocity v is perpendicular to the rod. After the collision, mass m_2 has velocity $-\frac{1}{2}\mathbf{v}$ as shown in Figure II. Answer the following in terms of the symbols given.

a. Using the principle of conservation of linear momentum, determine the velocity **v'** of the center of mass of this rod after the collision.

$$m_2 v = M_1 v' + \left(-\frac{v}{2}\right)\cdot m_2$$

$$\frac{3}{2}\frac{m_2 v}{M_1} = v'$$

b. Using the principle of conservation of angular momentum, determine the angular velocity ω of the rod about its center of mass after the collision.

Ball $L_{before} = L_{after} \leftarrow$ Ball and rod

$P_{ball}\cdot r = P'_{ball}\cdot r + I\omega$

from CM of stick to impact point

$$m_2 v\cdot\frac{L}{3} = -m_2\frac{v}{2}\cdot\frac{L}{3} + \frac{M_1L^2}{12}\omega$$

$$\frac{m_2 v}{2}\cdot L = \frac{m_1 L^2}{12}\omega$$

$$\frac{6\, m_2 v \cancel{L}}{L^{\cancel{2}} m_1} = \omega$$

c. Determine the change in kinetic energy of the system resulting from the collision.

$$KE_{before} = KE^{trans}_{ball} = \frac{1}{2}m_2 v^2$$

$$KE_{after} = KE^{trans}_{ball} + KE^{trans}_{rod} + KE^{rot}_{rod}$$

$$= \frac{1}{2}m_2\frac{v^2}{4} + \frac{1}{2}m_1 v'^2 + \frac{1}{2}I\omega^2$$

Plug in previous answers.

$$= \frac{1}{8}m_2 v^2 + \frac{1}{2}\cdot\cancel{m_1}\cdot\frac{9}{4}\frac{m_2^2}{m_1^{\cancel{2}}}v^2 + \frac{1}{2}\cdot\frac{1}{12}\cancel{m_1}\cancel{L^2}\cdot\frac{36\,m_2^2\cdot v^2}{\cancel{L^2}\cdot m_1^{\cancel{2}}}$$

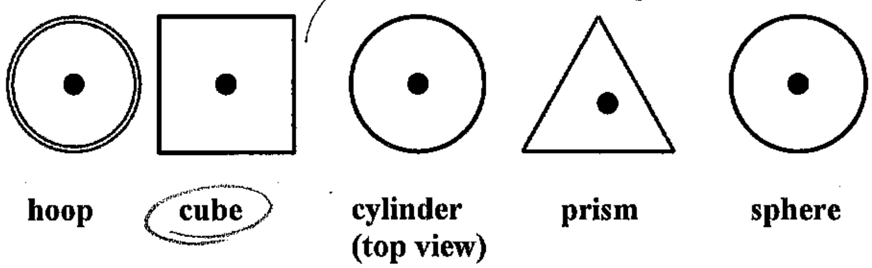
$$= \frac{1}{8}m_2 v^2 + \frac{9}{8}\frac{m_2^2}{m_1}v^2 + \frac{12}{8}\frac{m_2^2}{m_1}v^2$$

A mess! Nothing to be gained by doing more of this algebra.

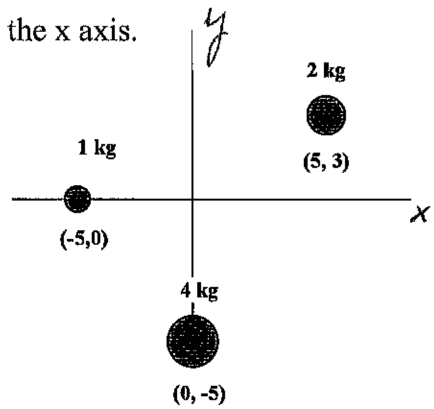
Short answer/multiple choice

1. Five solids with identical masses are shown below in cross section. Each has the same width at its widest part and is rotated through the center axis as shown by the dot. Which probably has the greatest moment of inertia? The least?

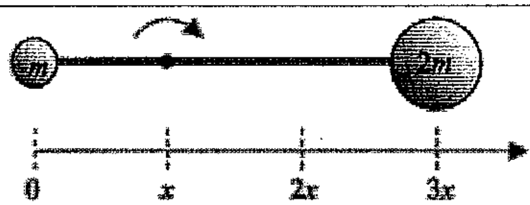
consistently largest radius so more mass is further from the center



2. Calculate the moment of inertia of the following system of point masses around the x axis.



$$I_{Tot} = I_1 + I_2 + I_4$$
$$= m_1 r_1^2 + m_2 r_2^2 + m_4 r_4^2$$
$$= (1\,kg)(0)^2 + (2\,kg)(3)^2 + (4\,kg)(5)^2$$
$$= 2 \cdot 9 + 4 \cdot 25$$
$$= 18 + 100$$
$$= 118$$



3. A solid sphere of mass *m* is fastened to another sphere of mass *2m* by a thin rod with a length 3*x*. The spheres have negligible size compared to *x* and the rod has negligible mass. What is the rotational mass of the system of spheres as the rod is rotated about the point located at position *x*, as shown?

(A) 3 mx^2 (B) 4 mx^2 (C) 5 mx^2
(D) 9 mx^2 (E) 10 mx^2

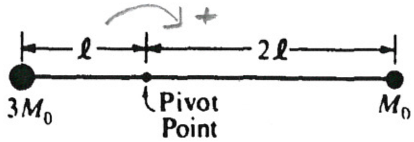
$$I_{Tot} = m_1 r_1^2 + m_2 r_2^2$$
$$= mx^2 + 2m(2x)^2$$
$$= mx^2 + 2 \cdot 4mx^2$$
$$= mx^2 + 8mx^2$$
$$= 9mx^2$$

$\Sigma \tau = I\alpha$

$-3M_o g \ell + M_o g 2\ell = \alpha\left(3M_o \ell^2 + M_o (2\ell)^2\right)$

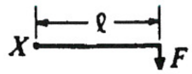
$-M_o g \ell = \alpha\left(7 M_o \ell^2\right)$

$\frac{-g}{7\ell} = \alpha$



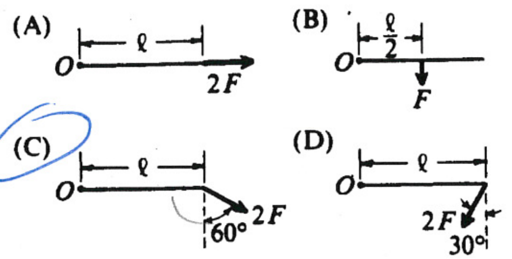
8. A light rigid rod with masses attached to its ends is pivoted about a horizontal axis as shown above. When released from rest in a horizontal orientation, what is the angular acceleration of the object? (Hint: what causes the torque and what is the rotational mass?)

(A) $\frac{g}{7l}$ (B) $\frac{g}{5l}$ (C) $\frac{g}{4l}$ (D) $\frac{5g}{7l}$



$\tau = F\ell$

9. In which of the following diagrams is the torque about point O equal in magnitude to the torque about point X in the diagram above?



A) $\tau_A = \ell\, 2F \sin(180) = 0$

$\tau_B = F\frac{\ell}{2}$

$\tau_C = 2F\ell \sin(150) = 2F\ell\left(\frac{1}{2}\right) = F\ell$

$\tau_D = 2F\ell\sin(60) = 2F\ell \cdot 0.866 = 1.7F\ell$

A wheel with rotational inertia I is mounted on a fixed, frictionless axle. The angular speed ω of the wheel is increased from zero to ω_f in a time interval T.

$\alpha = ?$; $\omega_o = 0$, $\omega_f = \omega_f$, $t = T$

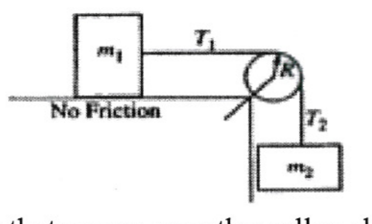
$\omega_f = \cancel{\omega_o}^{\,0} + \alpha t$

$\frac{\omega_f}{T} = \alpha$

10. What is the average net torque on the wheel during this time interval?

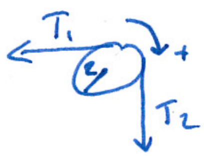
(A) $\frac{\omega_f}{T}$ (B) $\frac{I\omega_f^2}{T}$ (C) $\frac{I\omega_f}{T^2}$ (D) $\frac{I\omega_f}{T}$

$\tau = I\alpha = I\frac{\omega_f}{T}$



11. Two blocks are joined by a light string that passes over the pulley shown above, which has radius R and moment of inertia I about its center. T_1 and T_2 are the tensions in the string on either side of the pulley and α is the angular acceleration of the pulley. Which of the following equations best describes the pulley's rotational motion during the time the blocks accelerate?

(A) $m_2 gR = I\alpha$ (B) $T_2 R = I\alpha$ (C) $(T_2 - T_1)R = I\alpha$ (D) $(m_2 - m_1)gR = I\alpha$



$\Sigma \tau = I\alpha$

$T_2 R - T_1 R = I\alpha$

$(T_2 - T_1)R = I\alpha$

12. A bowling ball of mass M and radius R. whose moment of inertia about its center is (2/5)MR^2, rolls without slipping along a level surface at speed v. The maximum vertical height to which it can roll if it ascends an incline is

(A) $\frac{v^2}{5g}$ (B) $\frac{2v^2}{5g}$ (C) $\frac{v^2}{2g}$ (D) $\frac{7v^2}{10g}$

$KE_r + KE_T = GPE$

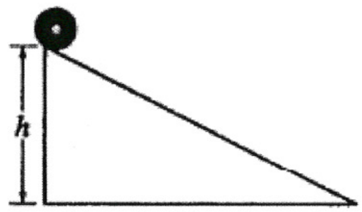
$\frac{1}{2}I\omega^2 + \frac{1}{2}mv^2 = mgh$

$\cancel{\frac{1}{2}}\left(\frac{2}{5}\cancel{M}\cancel{R^2}\right)\left(\frac{v^2}{\cancel{R^2}}\right) + \frac{1}{2}Mv^2 = \cancel{M}gh$

$\frac{1}{5}v^2 + \frac{1}{2}v^2 = gh$

$h = \frac{7v^2}{10g}$

Questions 13-14



A sphere of mass M, radius r, and rotational inertia I is released from rest at the top of an inclined plane of height h as shown above.

13. If the plane is frictionless, what is the speed v_{cm}, of the center of mass (linear speed) of the sphere at the bottom of the incline?

(A) $\sqrt{2gh}$ (B) $\frac{2Mghr^2}{I}$ (C) $\sqrt{\frac{2Mghr^2}{I}}$ (D) $\sqrt{\frac{2Mghr^2}{I + Mr^2}}$

No friction means it can't roll, it slides

14. If the plane has friction so that the sphere rolls without slipping, what is the speed v_{cm} (linear speed) of the center of mass at the bottom of the incline?

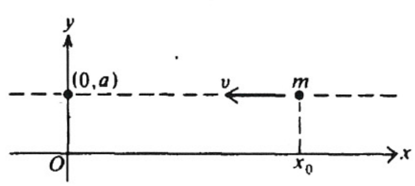
(A) $\sqrt{2gh}$ (B) $\frac{2Mghr^2}{I}$ (C) $\sqrt{\frac{2Mghr^2}{I}}$ (D) $\sqrt{\frac{2Mghr^2}{I + Mr^2}}$

$GPE = KE_R + KE_T$

$mgh = \frac{1}{2}I\omega^2 + \frac{1}{2}mv^2$

$mgh = \frac{v^2}{2}\left(\frac{I}{R^2} + m\right)$

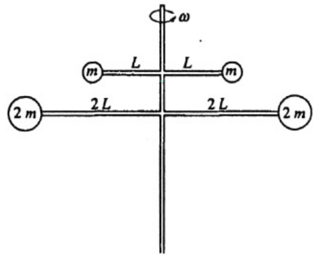
$\frac{2mghR^2}{I + mR^2} = v^2$



15. A particle of mass m moves with a constant speed v along the dashed line y = a. When the x-coordinate of the particle is x_o, the magnitude of the angular momentum of the particle with respect to the origin of the system is

(A) zero (B) mva (C) mvx_o (D) $mv\sqrt{x_0^2 + a^2}$

Questions 16 and 17



The rigid body shown in the diagram above consists of a vertical support post and two horizontal crossbars with spheres attached. The masses of the spheres and the lengths of the crossbars are indicated in the diagram. The body rotates about a vertical axis along the support post with constant angular speed ω.

16. If the masses of the support post and the crossbars are negligible, what is the ratio of the angular momentum of the two upper spheres to that of the two lower spheres?
(A) 2/1 (B) 1/2 (C) 1/4 (D) 1/8

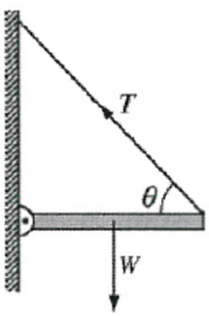
mass $\times 2$

$R \times 2$

$I = mr^2$

17. What is the moment of inertia of the system?
(A) 2 mL^2 (B) 6 mL^2 (C) 9 mL^2 (D) 10 mL^2 (E) 18 mL^2

$2\left(2m\cdot(2L)^2\right) + 2\left(mL^2\right)$



18. A uniform beam of weight W is attached to a wall by a pivot at one end and is held horizontal by a cable attached to the other end of the beam and to the wall, as shown above. T is the tension in the cable, which makes an angle θ with the beam. Which of the following is equal to T?

(A) $\dfrac{W}{2\cos\theta}$

(B) $\dfrac{W}{2\sin\theta}$

(C) $\dfrac{W}{\cos\theta}$

(D) $\dfrac{W}{\sin\theta}$

(E) W

$\tau_T = \tau_W$

$TL\sin\theta = W\cdot \frac{L}{2}$

$T = \dfrac{W}{2\sin\theta}$